

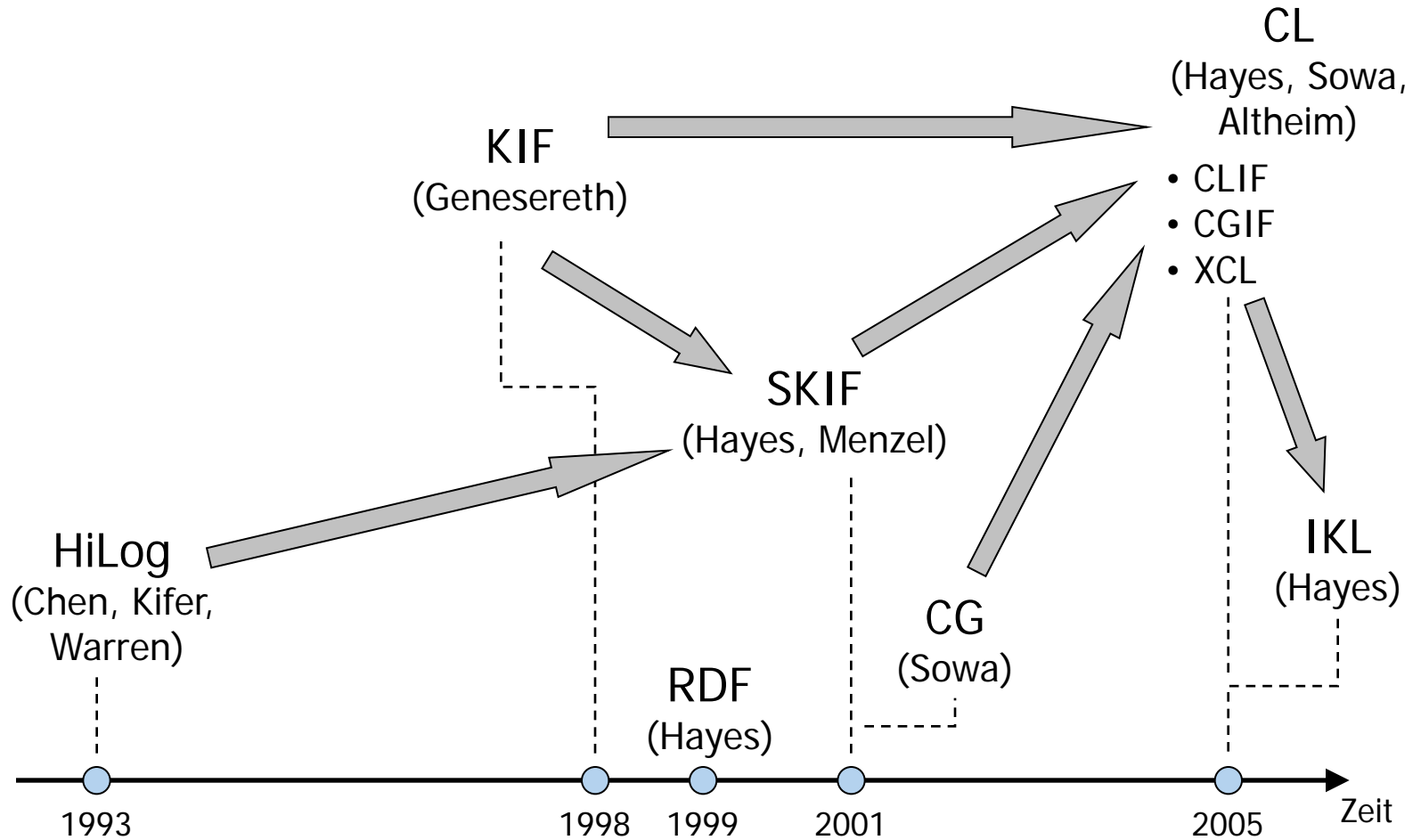
# Common Logic

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# Background



# The Common Logic dialects

- Common Logic is a family of languages
- Common Logic dialects:
  - Common Logic abstract syntax
  - Common Logic Interchange Format (CLIF)
  - Conceptual Graph Interchange Format (CGIF)
  - Extended Common Logic Markup Language (XCL)
- All have the same semantic

# CL abstract syntax categories

- (text, phrase, comment, module, importation, irregular sentence)
- sentence
- quantified sentence
- boolean sentence
- atom
- term
- functional term
- term sequence
- name

# CL abstract syntax categories

- quantified sentence:
  - quantification over a finite sequence of names or sequence names
- boolean sentence:
  - arity of conjunction and disjunction is arbitrary
  - empty conjunction is the „true“ element
  - empty disjunction is the „false“ element
- name:
  - no distinction between relation names, function names and individual names
- sequence name:
  - stands for an arbitrary sequence of arguments

# CL semantics

- **vocabulary** is a set of names and sequence names
- **interpretation**  $I$  of a vocabulary  $V$ :
  - a set  $UR_I$  with a distinguished nonempty subset  $UD_I$
  - and four mappings
    - $rel_I : UR_I \rightarrow Pow ( UD_I^* )$
    - $fun_I : UR_I \rightarrow Pow ( UD_I^* \times UD_I )$
    - $int_I : \text{from names in } V \rightarrow UR_I$
    - $seq_I : \text{from sequence names in } V \rightarrow UD_I^*$
- $UR = \text{universe of reference, } UD = \text{universe of discourse}$
- $int_I(x) \in UD_I$  if and only if  $x$  is a discourse name

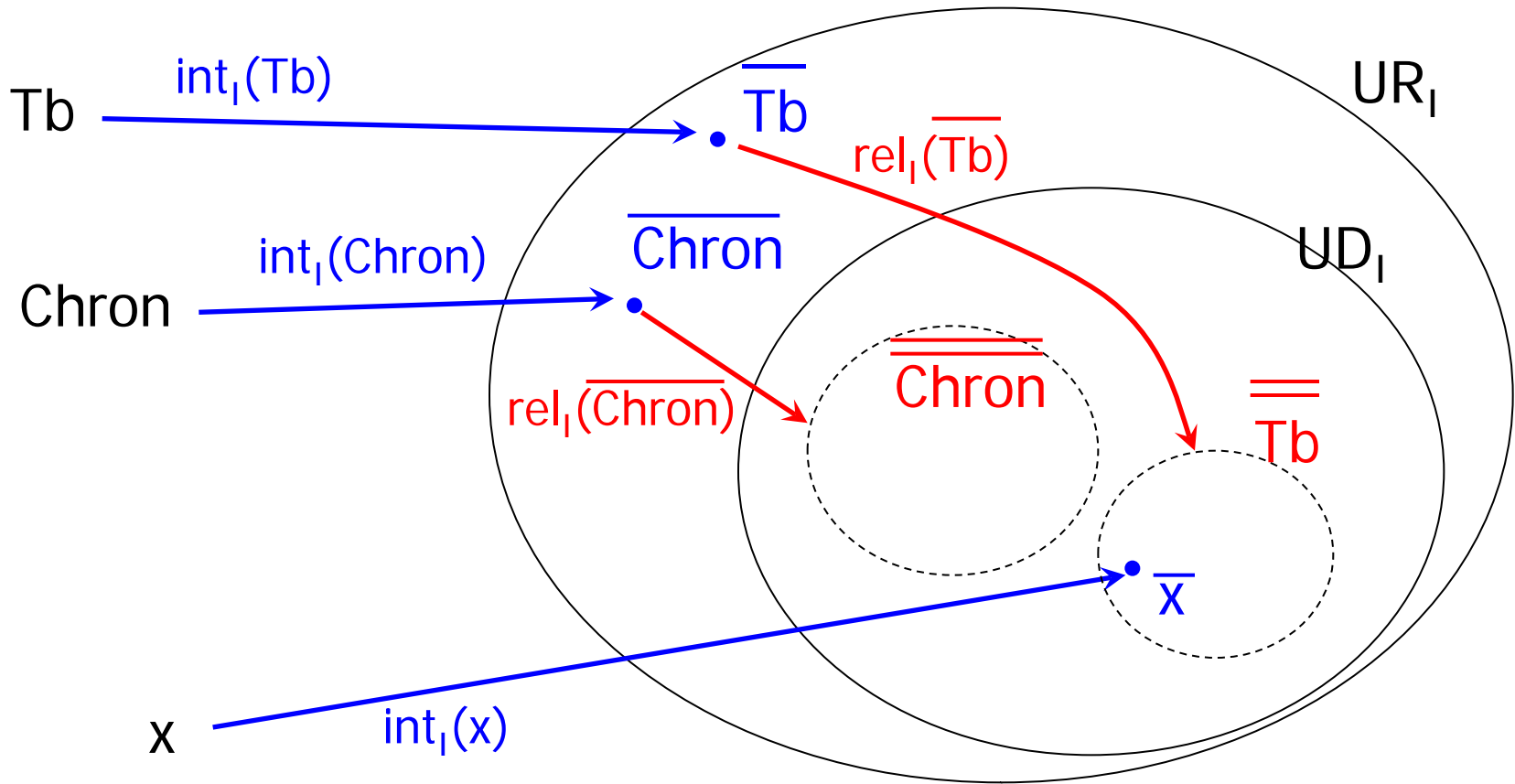
# Examples (in CLIF syntax)

- (not (exists (x) (and (Tb x) (Chron x))))
  - $I((Tb\ x))$  true if  $I(x) \in rel_1(I(Tb))$
  - $I((Tb\ x))$  true if  $int_1(x) \in rel_1(int_1(Tb))$
  - $I((Chron\ x))$  true if  $I(x) \in rel_1(I(Chron))$
  - $I((Chron\ x))$  true if  $int_1(x) \in rel_1(int_1(Chron))$



# Examples

(not (exists (x) (and (Tb x) (Chron x))))

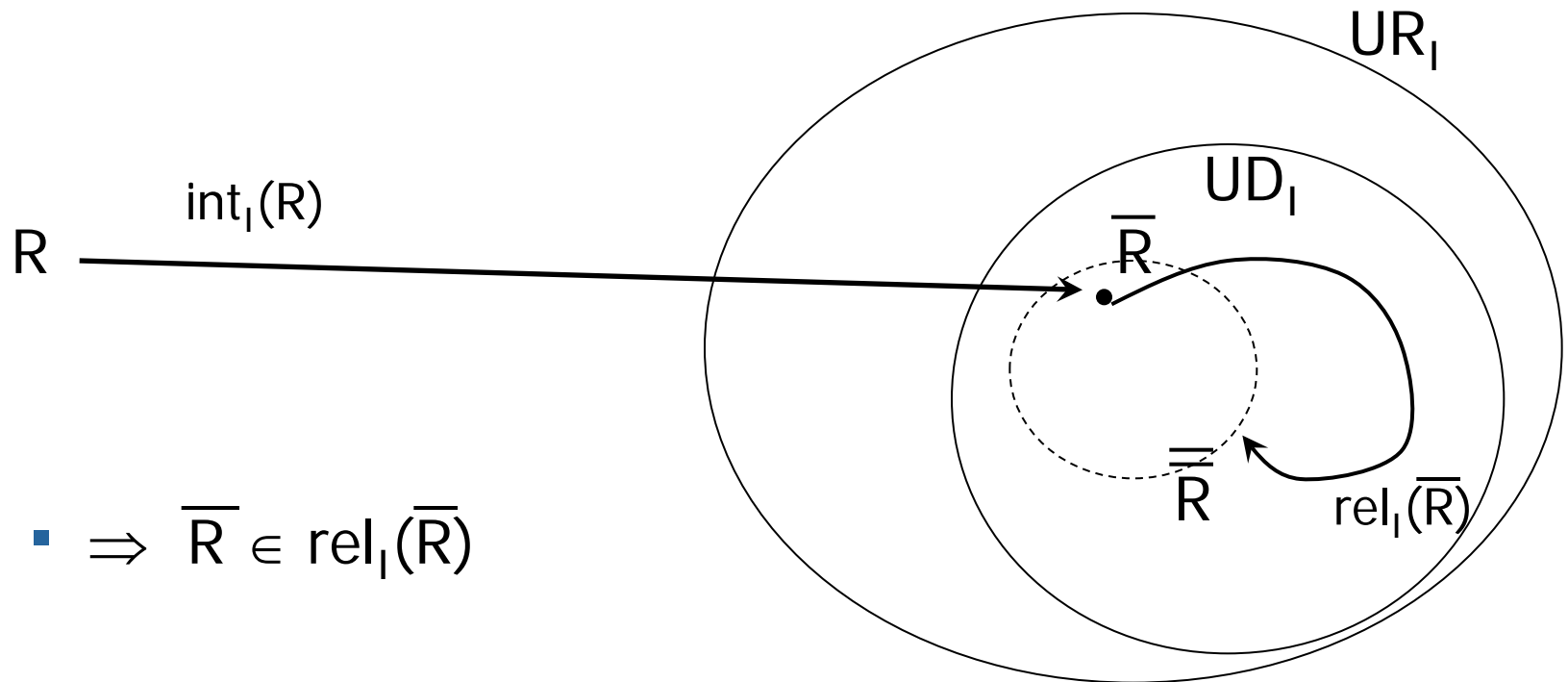


# Examples

- $(\text{forall } (x) (\text{if } (\text{Chron } x) (\text{and } (\text{exists } (u) (\text{lb } u \ x)) (\text{exists } (v) (\text{rb } v \ x))))))$ 
  - $I((\text{lb } u \ x))$  true if  $(I(u), I(x)) \in \text{rel}_1(I(\text{lb}))$
  - $I((\text{lb } u \ x))$  true if  $(\text{int}_1(u), \text{int}_1(x)) \in \text{rel}_1(\text{int}_1(\text{lb}))$
- $(\text{sum } a \ b)$ 
  - $I((\text{sum } a \ b)) = \text{the } x \text{ such that } (I(a), I(b), x) \in \text{fun}_1(I(\text{sum}))$
  - $I((\text{sum } a \ b)) = \text{the } x \text{ such that } (\text{int}_1(a), \text{int}_1(b), x) \in \text{fun}_1(\text{int}_1(\text{sum}))$

# Uncommon Aspects

- self-application:  $(R R)$ 
  - $I((R R))$  true if  $I(R) \in \text{rel}_I(I(R))$
  - $I((R R))$  true if  $\text{int}_I(R) \in \text{rel}_I(\text{int}_I(R))$



- $\Rightarrow \bar{\bar{R}} \in \text{rel}_I(\bar{R})$

# Uncommon Aspects (self-application)

- Can you construct the Russell paradox ?
- Some considerations:
  - $\forall x C(x) \leftrightarrow P(x) \wedge Q(x)$ 
    - Interpretation:
      - $I(x) = \text{int}_1(x)$ , let be  $\text{int}_1(x) = \bar{x}$
      - $I(C) = \text{rel}_1(\text{int}_1(C))$ , let be  $\text{rel}_1(\text{int}_1(C)) = \bar{\bar{C}}$ ,  
 $I(P) = \bar{\bar{P}}$  and  $I(Q) = \bar{\bar{Q}}$
      - $\bar{x} \in \bar{\bar{C}} \leftrightarrow \bar{x} \in \bar{\bar{P}} \wedge x \in \bar{\bar{Q}} \leftrightarrow \bar{x} \in \bar{\bar{P}} \cap \bar{\bar{Q}}$   
and so:  $\bar{\bar{C}} = \bar{\bar{P}} \cap \bar{\bar{Q}}$

# Uncommon Aspects (self-application)

- $\forall X D(X) \leftrightarrow X(X)$

– Interpretation:

- like before
- Is this the correct meaning ?

$$\forall \bar{x} \forall \bar{y} \bar{y} \in \text{rel}_1(\bar{D}) \leftrightarrow \bar{y} \in \text{rel}_1(\bar{X}) \wedge \bar{x} = \bar{y}$$

# Uncommon Aspects

- sequence names:
  - universal quantifier binding a sequence name has the same semantic import as the infinite conjunction of all the expressions obtained by replacing the sequence name by a finite sequence of names, all bound by the universal quantification
  - Example: (in CLIF syntax)  
`(forall (...x) (foo ...x))`  
has the same meaning as an infinite conjunction of this form  
`(and (foo`  
`(forall (x1) (foo x1))`  
`(forall (x1 x2) (foo x1 x2))`  
`(forall (x1 x2 x3) (foo x1 x2 x3))`  
`... )`